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Kinetic Energy and Momentum Considerations to Water Drop Impact

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SUMMARY:

Physically-based analyses of water drop impact are possible using momentum and impulse. The dynamic response, rise time and sampling rate requirements for impulse measurement using piezoelectric sensors are analyzed. The ratio of measured impulse to drop momentum is determined for three surface conditions.

KEYWORDS:

droplet, impact, impulse, kinetic energy, momentum

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Abstract

The impulse applied to a surface from water drop impact can be measured experimentally by recording and integrating impact force over time. A momentum number, defined as measured impulse divided by the drop's momentum, is a dimensionless physically-based parameter which can be used for modeling soil movement due to impact. The momentum number value is one for a smooth-dry surface and is greater than one for a smooth-wet or a rough surface. Accurate impulse measurements do not require sensors with fast dynamic response or calibration, but resonant frequencies must be sufficient to minimize signal amplification. Factors to be considered when measuring impulse or work from water drop impact are identified.

Introduction

The impact of falling water drops from rainfall or sprinkler irrigation systems has become an important research topic due to its destructive effects on the surface structure of soils. The analysis of a water drop striking a soil surface can be separated into two areas: the fluid mechanics of the water drop during impact and the mechanical properties of soil under impact loading. The objective of this paper is to analyze the fluid mechanics of water drops striking a rigid surface using a whole-drop approach and the principles of kinetic energy, work, momentum and impulse. Impact sensor frequency response and data acquisition sampling rate requirements are also analyzed.

Some researchers (Bisal, 1960; Bubenzer and Jones, 1971; Rose, 1960; Wischmeier and Smith, 1958) have measured soil surface destruction parameters such as soil splashed, depth of soil crust, or reduction of infiltration rate and empirically calibrated it to a parameter that usually included drop size, velocity, momentum or kinetic energy. Other researchers (Imaynitov et al, 1966; Imeson et al, 1981; Kinnell, 1972, 1976; Mikhaylovskaya, 1964) used empirically-based impact sensors that produced some form of output signal that varied with drop size and velocity. However, no conclusive independent parameter for the dynamic characteristics of a water drop at impact was determined from these empirically-based experiments. These experiments determined that drop size or mass and impact velocity are the most significant variables affecting water drop impact and subsequent soil surface destruction. Calibrations of soil parameters or sensor output to drop size, mass or velocity were generally of the form,

$$Y = a\, d^{b1}\, m^{b2}\, v^{b3} \qquad (1)$$

where,
Y = soil parameter or sensor output
d = drop diameter
m = drop mass
v = drop velocity
a,b1,b2,b3 = regression constants

Since kinetic energy and momentum are calculated from drop mass and velocity, they are a combined parameter for these variables, but have no specific significance to the output from an empirically-based impact sensor or to soil destruction parameters.

Physical Significance of Momentum and Kinetic Energy

Momentum and kinetic energy are significant because they are the result of integrating Newton's Second Law with respect to time or displacement, respectively, as follows,

$$F = \frac{d(mv)}{dt} \text{ , Newton's Second Law} \quad \ldots \quad 2$$

and integrating with respect to time,

$$\text{Impulse, } Ft = \Delta mv \text{, Momentum change} \quad \ldots \quad 3$$

or integrating with respect to displacement,

$$\int F\,ds = \int \frac{d(mv)}{dt}\,ds$$

$$= d(mv)\ v$$

$$= m\ v\ dv \text{, with constant mass}$$

$$\text{Work, } Fs = \Delta\frac{1}{2}mv^2 \text{, Kinetic Energy Change - Losses} \quad \ldots \quad 4$$

Consequently, impulse is applied to a surface due to a change in momentum, and work is applied due to a change in kinetic energy minus any energy losses such as inelastic deformation or increased water temperature. As independent parameters, momentum or kinetic energy have specific physically-based significance only if they are compared to measured impulse or work, respectively, that is applied to the impact surface.

A similar result is obtained from dimensional analysis of water drop impact. Considering only perpendicular impact with the surface and ignoring the viscosity and surface tension of water and the acceleration of gravity, the significant variables are

W = work, [FL]

I = impulse, [FT]

ρ = liquid density, $[FT^2L^{-4}]$

v = impact velocity, $[LT^{-1}]$

d = drop diameter, [L]

Dimensionless numbers are generated by combining these variables so that dimension is eliminated. The repeating variables used are; ρ to eliminate force, v to eliminate time, and d to eliminate length. Combining ρ, v and d with W and I individually results in,

$$\frac{W}{\rho v^2 d^3} \quad \text{and} \quad \frac{I}{\rho v d^3}$$

If viscosity, surface tension and acceleration of gravity were included in the dimensional analysis, the Reynolds, Weber, and Froude Numbers would have been generated, respectively.

Recognizing that the volume of a sphere equals $\Pi d^3/6$ and that mass equals density times volume, the two dimensionless numbers are proportional to

$$\frac{W}{\frac{1}{2}mv^2} \quad \text{and} \quad \frac{I}{m\ v}$$

which are just the ratios of work to kinetic energy, and impulse to momentum, respectively. They will be referred to as Kinetic Energy and Momentum Numbers, respectively. If either work or impulse can be measured, then a physically-based analysis of these factors to kinetic energy or momentum, respectively can be conducted instead of empirically-based analyses.

Schleusener and Kidder (1960) measured the work done by water drops to deflect a target mounted on a cantilever beam. They were able to measure the strain energy stored in the beam, which is that portion of the drops' kinetic energy that is converted to work (applied force times the displacement of the impact target), which was then stored in the beam as strain energy.

Methods of Impulse Measurement

Impulse, impact velocity and drop mass can be measured so that a physically-based impulse and momentum analysis can be made of water drop impact. The impulse due to water drop impact was measured for three different surface situations. Impact forces were measured with a Kistler[1] model 9712A5 piezoelectric force transducer (impact sensor) which has a resonant (no load) frequency of 70 kHz, a resolution of 0.89 mN and a linearity of ±1%. This sensor can be calibrated statically with a known weight and the calibration is 4.50 N/volt ±0.08 N/volt standard deviation.

Circular impact plates of 15 to 20 mm diameter were mounted on this sensor to simulate different surface conditions. A smooth surface plate was used for two surface conditions; one with the surface dry and the other with residual water remaining on the surface from previous water drop impacts. A second plate with gravel particles bonded to the plate was used to simulate a rough surface condition. The gravel particles were those sieved out between Tyler Standard Screen No. 7 and 8, and which have an average particle size of 2.58 mm. Residual water also remained on the gravel plate during impact measurements.

1. Note: The mention of trades names or commercial products does not constitute their endorsement or recommendation for use by the USDA-ARS.

The impact plates were mounted on the sensor with a small bolt. The additional mass of the bolt and an impact plate further reduces the resonant frequency of the sensor. The surfaces of piezoelectric sensors exhibit negligible displacement due to impact and are considered essentially rigid. However, they are still treated as a spring-mass system. The resonant frequency of an oscillating system of the first order with no damping (Derrick and Grossman, 1976) is calculated by

$$f_r = \frac{1}{2\pi}(C/m)^{1/2} \qquad 5$$

where, f_r = resonant frequency, kHz

C = spring stiffness, N/μm

m = oscillating mass, kg

The Kistler 9712A5 sensor has a "rigidity" or spring stiffness of approximately 875 N/μm, so its oscillating mass without any plate or the mounting bolt is approximately 4.5 g. The bolt mass is 0.7 g and the mass of the plates were less than 2.1 g each to keep the combined mass less than 7.3 g so that the resonant frequency of the system was always greater than 55 kHz.

The sensor was connected to and powered by Kistler's model 5112 piezotron coupler, which is AC coupled to reduce output signal noise. The coupler can have a distorting or filtering effect on the signal that is dependent upon the sensor driving current, cable capacitance and signal amplitude. For a 4 m cable that was used between the sensor and coupler, a 2 ma driving current, and up to 5 volts peak output, all signals less than 160 kHz are unaffected, so the output signal has no high-frequency filtering.

The maximum operating frequency for dynamic transducers is usually defined as the frequency at which signal amplitude is ≤ 5% of resonant frequency (Hugli, no date). For quartz piezoelectric transducers, this maximum frequency is 22% of resonant frequency. Therefore, the maximum frequency of force loading on the sensor is 12 kHz which has a period of 83 μs. The maximum force loading rate from water drop impact occurs from initial impact to the time of peak force, is sinusoidal shaped, occurs in 13 to 22 microseconds, and specifically 19 μs for a 3.83 mm diameter drop falling at 8.75 m/s (Nearing et al, 1986). This loading approximates the first quarter period of a sine wave, so the sensor can measure water drop impact with times to peak force ≥ 21 μs. Therefore, 3.83 mm water drops falling from 1.5 m with an impact velocity of 5.00 m/s were used which should exhibit times to peak force ≥ 21 μs.

The 3.83 mm drops were produced with a 16 guage syringe needle with the tip cut squarely, and drop size and mass (29.4 mg) were determined by weighing a known number of drops. Drop velocity was measured by a double-ring electrostatic method (Hinkle et al, 1987). The minimum operating frequency is determined by the low-frequency time constant which for the sensor and coupler together is seven seconds, which only affects signals with frequencies less than one hertz.

The impact force signals were measured and recorded with a Cyborg Isaac 2000 analog-to-digital recorder connected to a microcomputer. Cyborg's

Discovery and Fastscanner software were used to control the Isaac 2000 recorder, subtract out any nonzero baseline, smooth the recorded signal and integrate the signal over time to obtain measured impulse. Experimental considerations, namely impact sensor high-frequency response, and the sampling rate of measured impact force and its integration over time will be analyzed in the following sections. An analysis of force integrated over displacement to obtain work would be similar and is not included.

Dynamic Response Analysis

In addition to loading piezoelectric sensors at frequencies sufficiently below the resonant frequency as just discussed, the rise time or the time to respond to a signal input is also an important dynamic characteristic which should be considered and analyzed. For Kistler sensors, the rise time is defined as the time for the output signal to increase from 10 to 90 percent of a step-increase input signal and is approximately 0.35 divided by the resonant frequency. The Kistler 9712A5 sensor has a rise time of approximately 6 microseconds. For water drop impact, greater rise times cause the output signal to have greater times to peak force and lower peak forces than the actual input signal. However, if impact forces are integrated over time, rise time has little effect on measured impulse as will be shown.

The force versus time relationship for drops as shown by Nearing et al (1986) were modeled by a linear rise to peak force, then an exponential decline to zero force, as follows,

$$f = kt, \qquad 0 \leq t \leq t_p \quad \text{. . . . . 6}$$

$$f = c + ae^{bt}, \qquad t > t_p \quad \text{. . . . . 7}$$

where, f = input force, [F]

t = time, [T]

t_p = time of peak force, [T]

a,b,c,k = constants.

The constant, c was set equal to -0.01 so that force would equal zero between 0.5 and 1 millisecond, and not be asymptotic.

The dynamic response of a first-order instrument is calculated from an equation for the input to the instrument and the following relationship,

$$T \frac{dy}{dt} + y = g(t) \quad \text{. . . . . 8}$$

where, y = sensor output

T = time constant, [T]

$g(t)$ = sensor input.

Equation 8 states that the time constant multiplied by rate of change of the sensor output plus sensor output equals sensor input. Time constants generally characterize the low-frequency baseline drift of sensors. The concept of a time constant can also characterize the high-frequency rise time.

For a step increase, g(t) in equation 8 equals a constant, y_c and the solution to equation 8 is

$$y/y_c = 1 - e^{-t/T} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad 9$$

Substituting the boundary conditions of 10% and 90% signal rise and with the time difference equal to the rise time results in the high-frequency time constant being approximately 50% of the rise time.

The output response of piezoelectric sensors determined by substituting eq. 6 and 7 for g(t) in equation 8 and solving, is as follows,

$$f_o = kt - kT + kte^{-t/T}, \quad \text{for } 0 \leq t \leq t_p \quad \ldots\ldots\ldots\ldots\ldots \quad 10$$

$$f_o = c + \frac{a}{Tb + 1}e^{tb} + e^{(t_p - t)/T}\left(kt_p - c - \frac{a}{Tb + 1}e^{t_p b}\right), \quad \text{for } t > t_p \quad . \quad 11$$

where, f_o = the output response (force) from the sensor, [F].

The output response for 2.5, 5, 10, 20, 40 and 80 microsecond rise times are shown in figure 1 along with output from eq. 6 and 7, with k = 0.0066, a = 1.13, b = -0.0075, and time to peak force is 15 microseconds. As rise times increase, recorded peak forces decrease and the times to peak force increase.

Actual impact forces do not peak so abruptly but are more sinusoidal shaped, so the sine function was substituted for g(t) in eq. 8, with the solution being,

$$f_o = \frac{\sin(ht) - hT\cos(hT) + hTe^{-t/T}}{1 + h^2T^2} \quad \ldots\ldots\ldots\ldots \quad 12$$

with h equal to $\pi/2$ divided by the time to peak force. The sine function and the output from eq. 12 for 2.5, 5, 10 and 20 microsecond rise times are shown in figure 2. The relative output peak forces and times to peak force are shown in Table 1. Actual output forces and times have values between those predicted by the two simulations. Piezoelectric sensors generally have rise times less than 10 µsec, so recorded peak forces may be up to 10% less and times to peak force 50% greater than that of the actual force signal. However, peak forces could be 5% greater due to signal frequencies near the 5% signal amplification limit as previously discussed. Actual changes of amplitude and time lag of the output signal can be determined from a dynamic calibration, generally done using a shock tube.

The simulated responses of measured impact forces for different rise times show that integrated impulse is virtually the same for rise times up to at least 80 µsec. Impulse was calculated by integrating equations 10 and

11 from time zero to the time when output force again equals zero. Calculated impulse for the six response curves in figure 1 decrease slightly with increasing rise times. All impulses are greater than 99.94 percent of the integrated impulse using input force equations 6 and 7. Consequently, the rise time of a piezoelectric sensor has little effect on measured impulse from water drop impact, even though peak force and time to peak force could be affected significantly. By a similar analysis, if impact forces can be integrated with respect to displacement, then work applied to a surface could be measured and the dynamic response of the sensor may also have little effect on integrated work.

Sampling Rate Requirements for Impulse Measurement

Even though sensors with short rise times are not required for measuring impulse, the sampling rate of the data acquisition system must be fast enough to minimize potential integration error. Assuming that measurement of the output signal from the sensor is not triggered by the output signal itself, integration can start at the last zero data point and end when force again equals zero. Using trapezoidal integration, the potential error of the integrated value can be significant if the sampling rate is too slow. An example of the integration error of equation 6 and 7 at a 10 kHz sampling rate for the two most extreme situations is shown in figure 3.

The maximum error of over-integrating occurs when the first non-zero data point is peak force. The maximum error of under-integrating occurs when the first zero end point is the time of impact. Integrations starting at all other times will have errors between the extreme over- and under-integration errors. The potential error is reduced if the ratio of sampling rate to signal frequency is increased. The same analysis was completed for faster sampling rates. Maximum potential integration errors, shown up to 5 percent, for various sampling rates and signal frequencies are shown in figure 4.

A general rule for data acquisition is that sampling rates should be at least twice the signal frequency. For any single integration of water drop impact forces over time, this rule will minimize the error to less than 0.5 % for signal frequencies greater than 30 kHz. However, measured times to peak force range from 13 to 22 μsec (Nearing et al, 1986), which are equivalent to signal frequencies of 11.4 to 19.2 kHz assuming the force loading to peak force is similar to the first quarter of a sine wave. For these slower signal frequencies, the sampling rate should be much greater than twice the signal frequency. Sampling rates greater than 60 kHz will minimize potential impulse integration error to less than 0.3% in the 11.4 to 19.2 kHz signal frequency range.

Impulse Measurement Results and Discussion

Measured impulse for the three different impact surface conditions are shown in table 2, all for 3.83 mm drops falling from 1.5 m height with a velocity of 5.00 m/s impact velocity. The sampling rate was 100 kHz. The standard deviations were determined from 10 replications per surface condition. Peak force and time to peak force could not be determined accurately due to some oscillation of the output signal near the time of peak force and were not recorded. This oscillation is due to slight vibration of the plate (Kistler, 1988). These oscillations had a period of

110 to 120 µsec, so the 100 kHz sampling rate was sufficient to record them. A sample force versus time recording showing the oscillation for water drop impact on the rough surface is shown in figure 5, along with the signal after smoothing. The software package was able to smooth the signal so that the end point of integration could be determined more accurately. The signals were smoothed by a factor equivalent to a 5 kHz low-pass filter which did not affect the integrated value as determined by tests with standard geometric-shaped signals.

A calculated momentum number of approximately one for the smooth dry surface shows that only the impulse caused by the drop's momentum is applied to the surface and that splash is essentially horizontal. A change in momentum perpendicular to the surface must equal the applied impulse because the water drops were impacted perpendicular to the surface and impact forces were measured perpendicular to the surface.

Momentum number values greater than one due to greater measured impulse for the smooth-wet surface and the rough surface show that additional forces are absorbed by these impact surfaces. These additional forces applied to the surface are caused by equal and opposite forces applied to the water by the surface that lifts the water up into a non-horizontal splash. The perpendicular component of the momentum of the splashed water should be equal to the additional impulse that is measured by the sensor.

The differences in applied impulse, and subsequently, the forces applied to the impact surface are caused by the differences in surface conditions. Thus, the earlier researchers doing empirically-based impact experiments with soils could not make definite conclusions about whether soil loss should be a function of the water drop's momentum or kinetic energy if they did not consider soil surface roughness. Their soil loss results should have had some correlation to surface roughness, splash angle and splash velocity, in lieu of measured impulse or work applied to their soil surfaces. Similar differences of measured work and kinetic energy number should also be expected for the three surface conditions. Applied impulse or work should be correlated more closely to soil loss than momentum or kinetic energy because impulse or work should better represent the total forces applied to the soil surface.

Sensor Limitations

A piezoelectric sensor can be used for measuring the impulse of whole drops at relatively slow impact velocities. Raindrops and sprinkler drops, though, may have velocities up to 9.1 m/s at sea level to 10.3 m/s at 2000 m elevation (Hinkle et al, 1987). Measuring impulse for water drops with these impact velocities would require a sensor with greater resonant frequency or less oscillating mass. Times to peak force for these velocities may be as short as 10 µs, which would require a resonant frequency of at least 114 kHz. Other sensors with greater resonant frequencies have smaller impact surfaces. Adding a larger diameter surface increases their oscillating mass which subsequently reduces their resonant frequency. Reducing the mass of the plate would require some reduction of the diameter of the plates which cannot be reduced much below 15 mm without missing some of the flow regime during impact, especially for impact angles less than perpendicular. So a trade-off exists between impact surface size and resonant frequency.

High-frequency response characterized by short rise times is not essential for determining impulse because the value of the integrated force signal is essentially unchanged over a range of rise times. However, rise time is inversely related to resonant frequency which is a limiting factor to measuring water drop impact forces. If actual peak forces and times to peak force are of concern, then the time-lag and peak signal decrease due to dynamic time-response (rise times > 0) need to be determined by a dynamic calibration.

Conclusions

Impulse measured from water drop impact and momentum calculated from measured impact velocity and drop mass enables a physically-based analysis of water drop impact. Piezoelectric sensors have sufficient dynamic response to accurately measure impulse and have impact surfaces large enough to receive the entire water flow regime. The potential error of integrating water drop impact forces over time is less than 0.3% if sampling rates are greater than 60 kHz for a typical range of water drop impact signal frequencies. If oscillations due to impact plate vibrations are present on the output signal, sampling rates should probably be at least 100 kHz.

A momentum number of one for the smooth-dry plate shows that the change of a water drop's momentum perpendicular to the plate is caused by an equal and opposite force between the water and the plate, as defined by Newton's Second Law. Momentum numbers greater than one for surfaces with residual water or roughness are a result of greater measured impulse due to additional impact forces needed to lift the water into a non-horizontal splash that has a perpendicular component of momentum. The momentum number should be a better basis for modeling soil loss parameters because it represents the physically-based relationship between drop size, velocity, and the total forces applied over the entire time of impact.

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Table 1. Simulated peak forces and times to peak force for water drop impact from a linear-exponential and sine functions.

	Time to Peak Force, μs		Peak Force, relative	
Rise time, μs	linear-exp	sine-wave	linear-exp	sine-wave
0	15	15	1.000	1.000
2.5	18	16.25	0.978	0.992
5	21	17.5	0.957	0.968
10	26	19.5	0.917	0.894
20	36	22	0.849	0.744

Table 2. Measured impulse and calculated momentum numbers for three impact surface conditions.

Impact surface	Impact Momentum, N-μs	Impact impulse, N-μs Mean	n	S.D.	Momentum Number
Smooth, dry	147.5	147.0	10	2.8	0.997
Smooth, wet	147.5	190.7	10	9.8	1.293
Rough, 2.58 mm average surface particle size	147.5	206.2	10	6.0	1.398

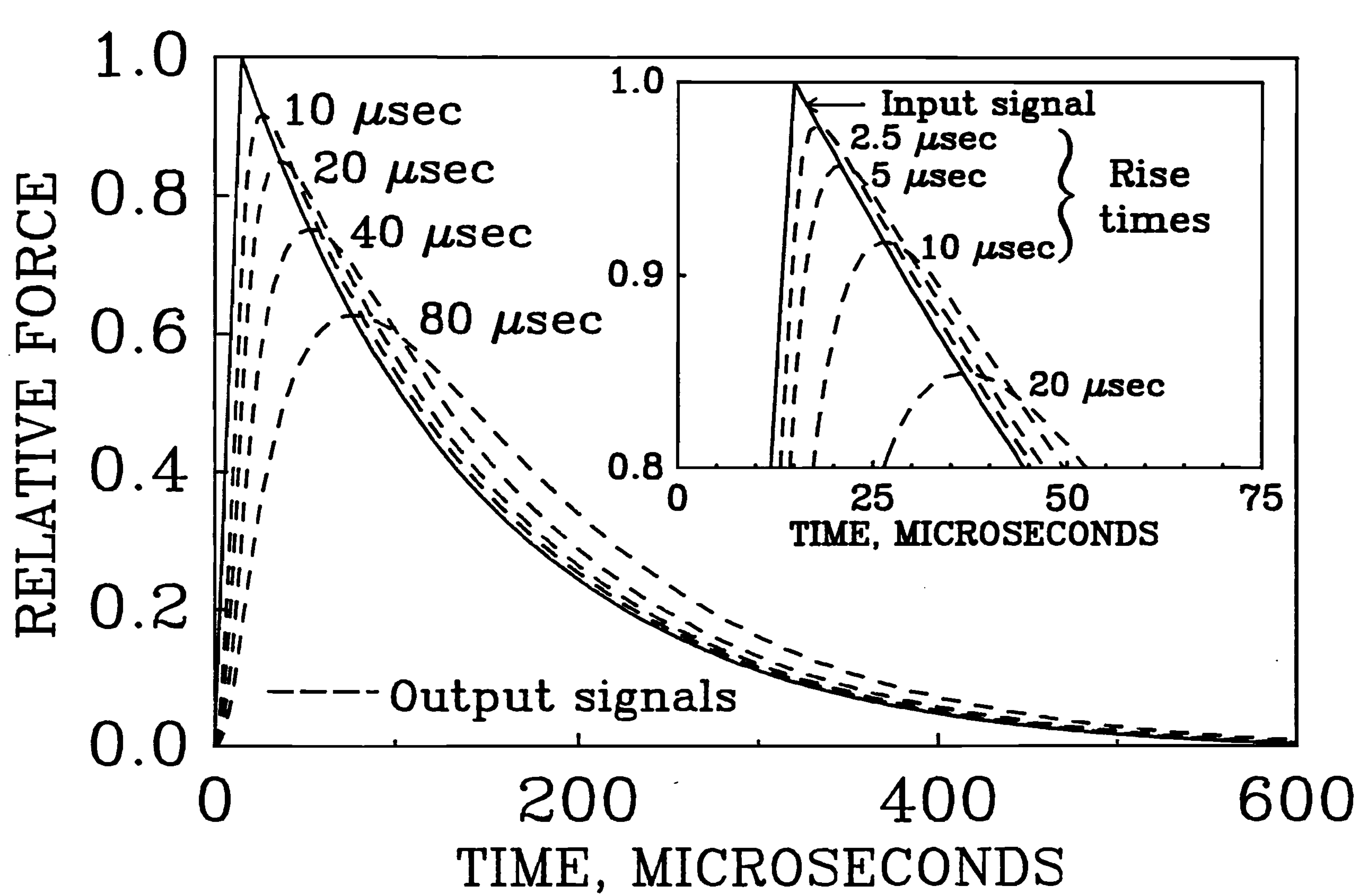

Figure 1. Simulated sensor output responses to a linear-exponential simulation of water drop impact forces.

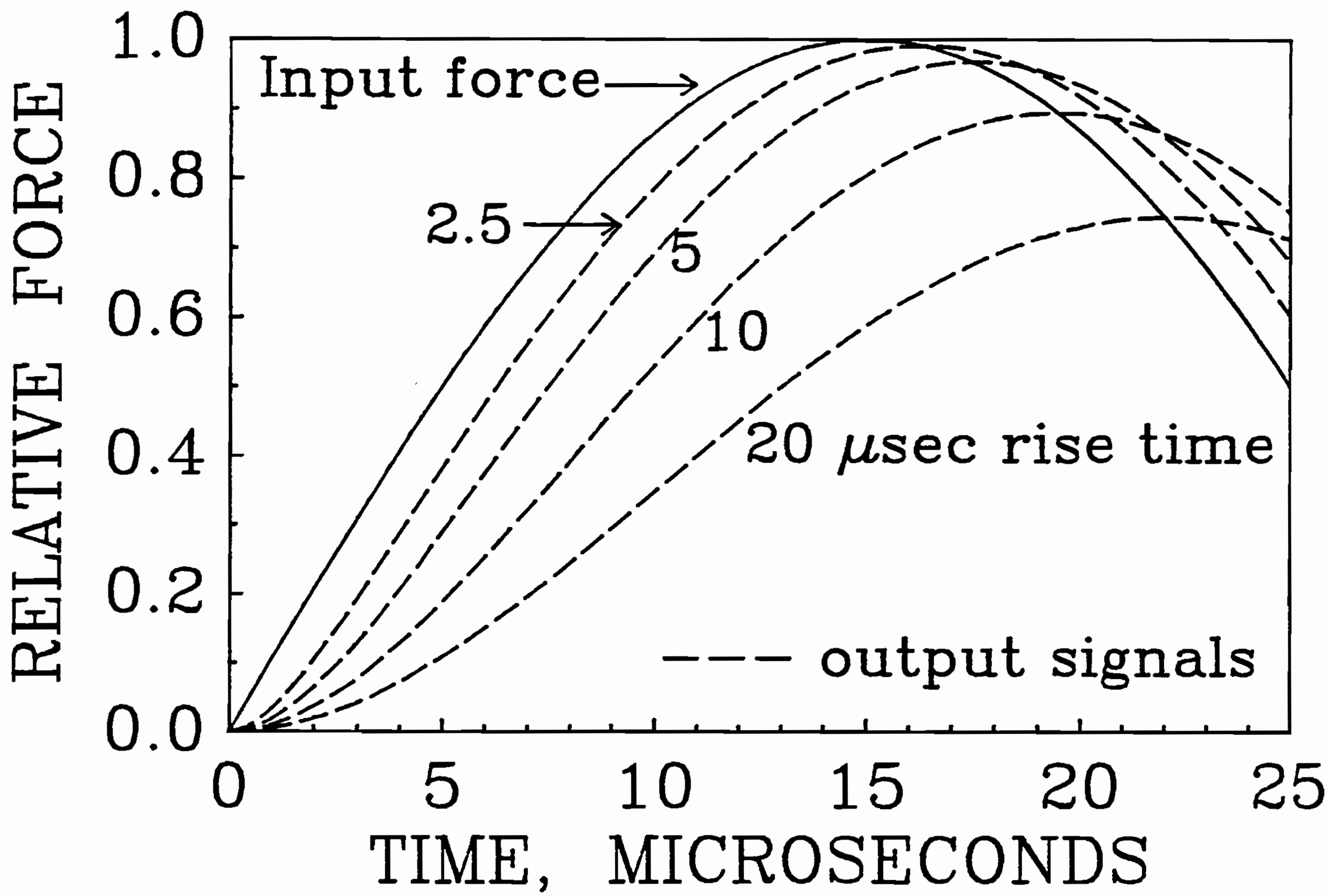

Figure 2. Simulated sensor output responses to a sine-wave simulation of water drop impact forces.

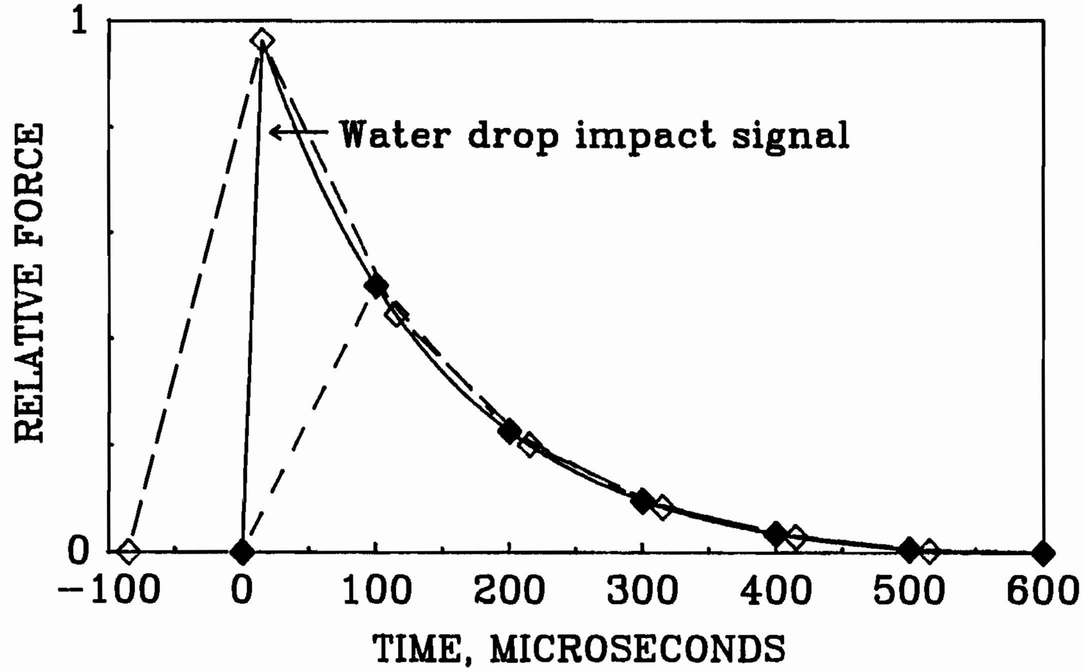

Figure 3. Graphical example of potential integration error of water drop impact forces.

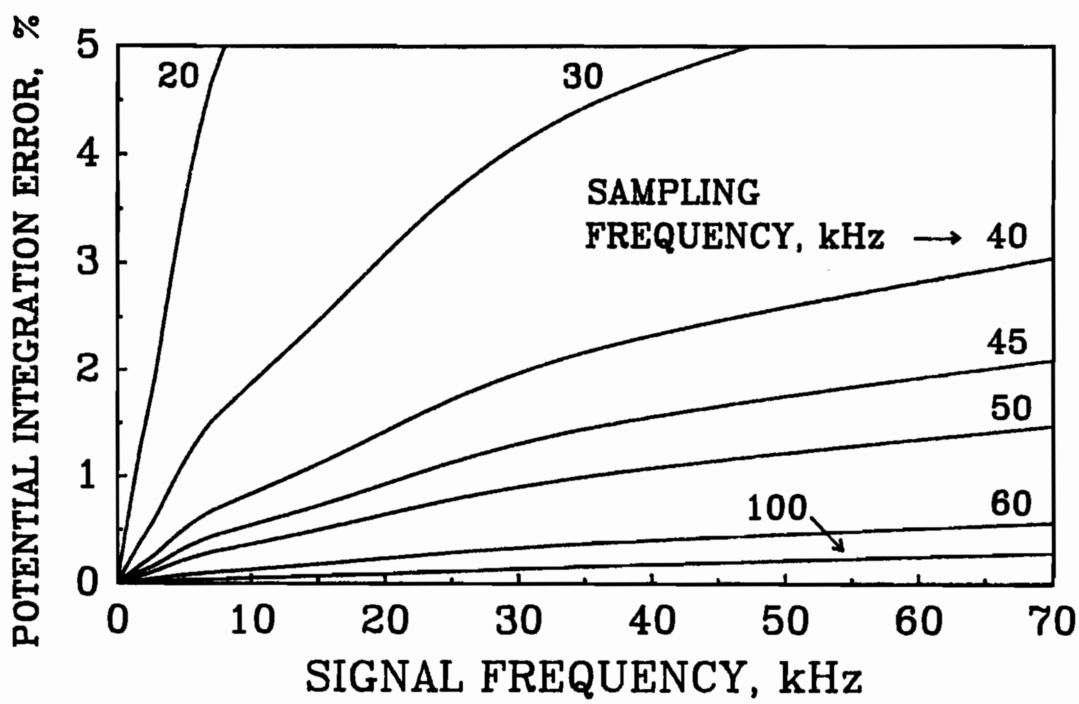

Figure 4. Potential error of integrating water drop impact forces over time.

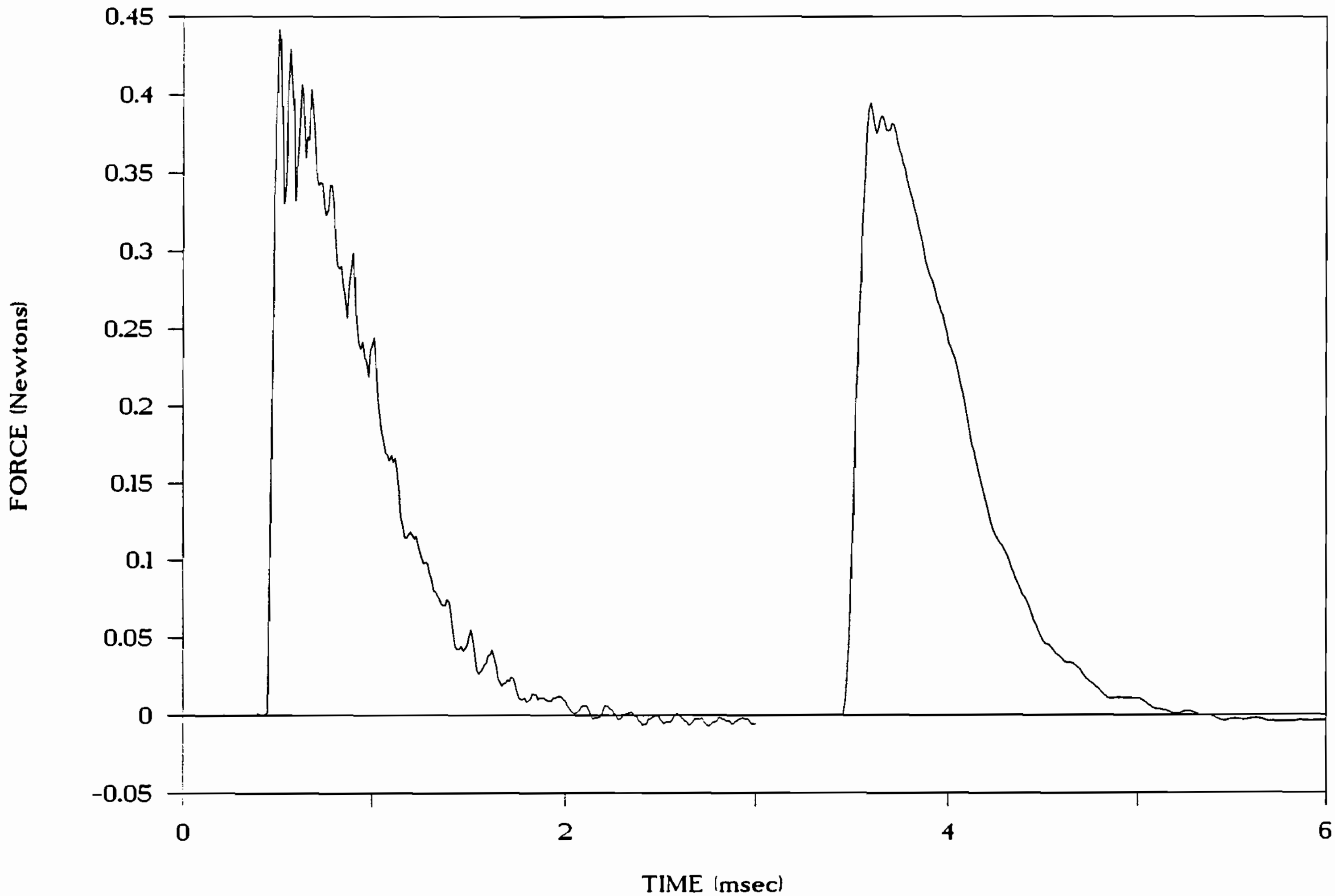

Figure 5. An example recording of the forces of a water drop impacting the rough surface with the recorded signal on the left and the same signal smoothed on the right.